

# Projective QMC — a new route to $T=0$

Karsten Held

MPI for solid state research, Stuttgart

*Hamburg, May 19, 2005*

- Motivation
- Projective quantum Monte Carlo (PQMC) method
- Results
  - One-band Hubbard model
  - Two-band Hubbard model
  - DCA (PQMC) study of the  $t$ - $t'$  Hubbard model
- Conclusion

# Thanks to...

M. Feldbacher, R. Arita — MPI Stuttgart

F. Assaad – Würzburg

Emmy Noether program of the DFG

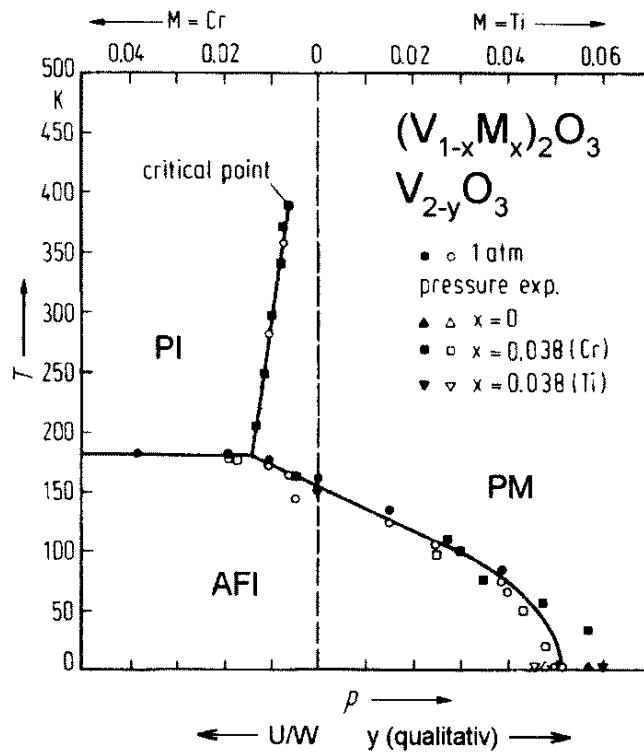
*Please ask questions...*

# Motivation

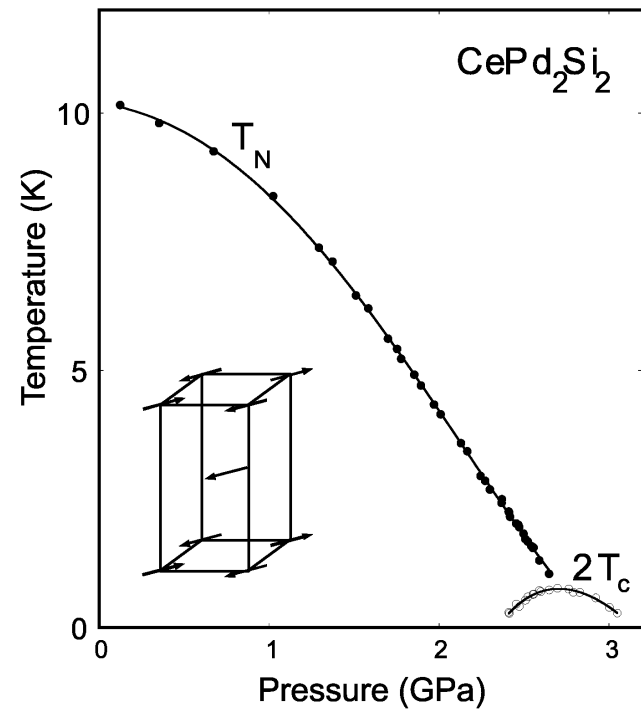
Realistic LDA+DMFT/quantum dot calculations

impurity problem ● many orbitals

● often interesting physics at low  $T$



not so low  $T$



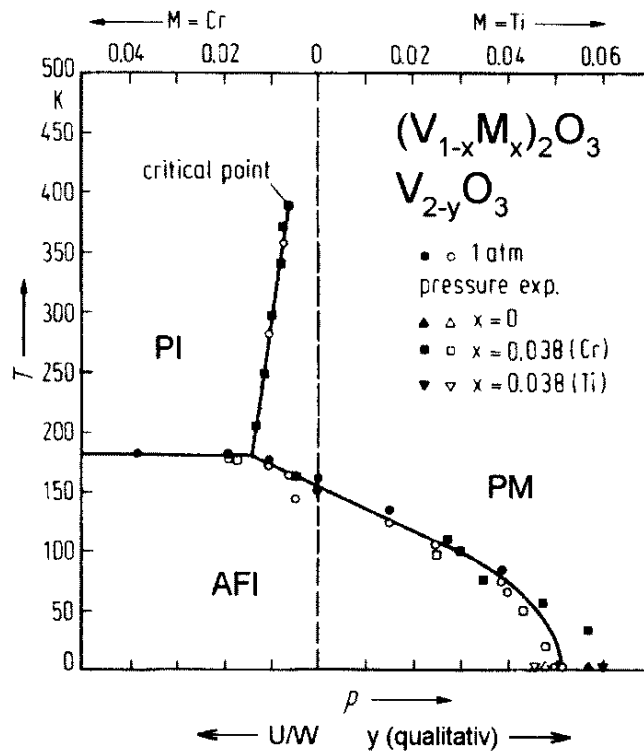
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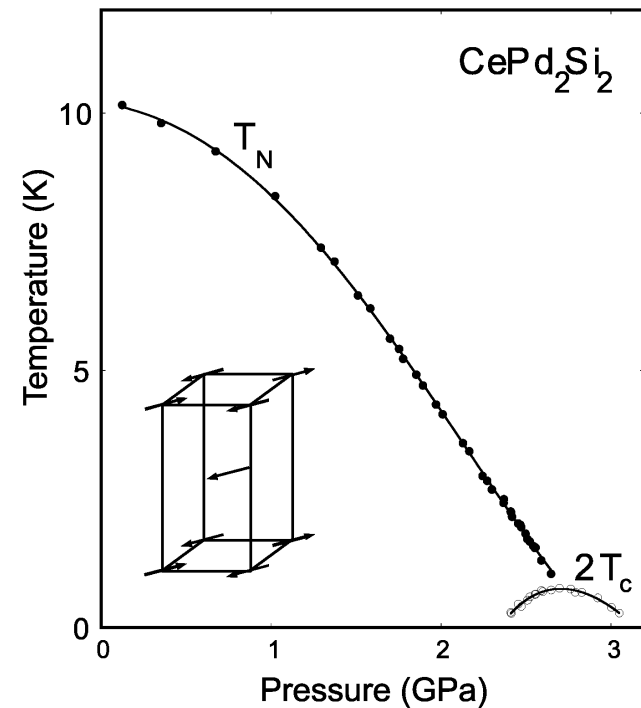
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very low  $T$

● Hirsch-Fye QMC  $T \gtrsim 300$  K

scaling  $\sim 1/T^3$

● Wilson's NRG  $\leq 2$  orbitals

scaling exponential in #orbitals

# Projective QMC for Anderson impurity model

Feldbacher, Held, Assaad PRL 93, 136405 (2004)

## Finite- $T$ Hirsch-Fye QMC (HF-QMC)

$$\langle \mathcal{O} \rangle = \frac{\text{Tr} \mathcal{O} e^{-\beta H}}{\text{Tr} e^{-\beta H}}$$

- thermal fluctuations
- effort  $\sim 1/T^3$

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$$\langle \mathcal{O} \rangle_{T=0} = \frac{\langle \Psi_{\text{GS}} | \mathcal{O} | \Psi_{\text{GS}} \rangle}{\langle \Psi_{\text{GS}} | \Psi_{\text{GS}} \rangle}$$

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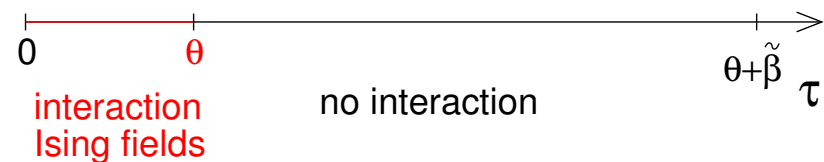
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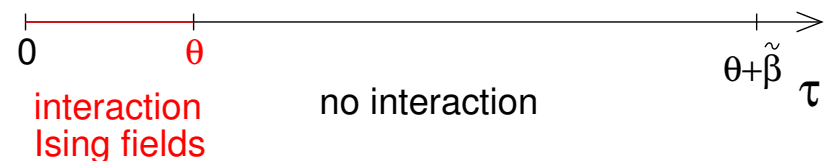
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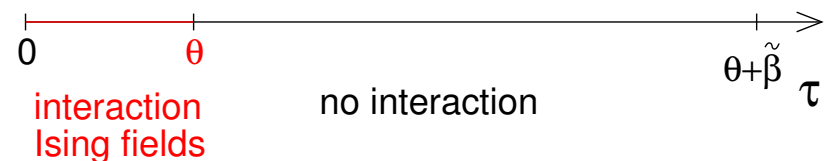
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extrapolation  $\theta \rightarrow \infty$  at the end

## In more detail...

We need Green function, i.e.,  $\mathcal{O} = -c(\tau_1)c^\dagger(\tau_2) = -e^{\tau_1/2 H} c e^{-(\tau_1-\tau_2)/2 H} c^\dagger e^{-\tau_2/2 H}$

**Green function matrix**

$$G(\tau_1, \tau_2) = - \lim_{\tilde{\beta} \rightarrow \infty} \frac{\text{Tre}^{-\tilde{\beta} H_0} e^{-(\theta-\tau_1)/2 H} c e^{-(\tau_1-\tau_2)/2 H} c^\dagger e^{-(\theta+\tau_2)/2 H}}{\text{Tre}^{-\tilde{\beta} H_0} e^{-\theta H}}$$

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**HF-QMC**

$\tilde{\beta} = 0$  instead of  $\lim_{\tilde{\beta} \rightarrow \infty}$

$\beta$  instead of  $\ominus$  physical meaning  $T$

**PQMC**

$\lim_{\tilde{\beta} \rightarrow \infty}$

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### HF-QMC

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$\beta$  instead of  $\ominus$  physical meaning  $T$

### PQMC

$\lim_{\tilde{\beta} \rightarrow \infty}$

$\lim_{\ominus \rightarrow \infty}$  at the end for  $T = 0$

### • Time discretization

$$e^{\ominus H} \text{ or } e^{\int_0^\ominus d\tau H} \longrightarrow e^{\Delta\tau \sum_{l=1}^L H}$$

### • Trotter decomposition

$$e^{\Delta\tau H_0 + \Delta\tau H_U} = e^{\Delta\tau H_0} e^{\Delta\tau H_U} + \frac{1}{2} \Delta\tau^2 [H_0, H_U] + \mathcal{O}(\Delta\tau^3)$$

### • Hubbard-Stratonovich decoupling

$$e^{\frac{\Delta\tau U}{2} (n_\uparrow - n_\downarrow)^2} = \frac{1}{2} \sum_{s_l = \pm 1} e^{\lambda s_l (n_\uparrow - n_\downarrow)} \quad \text{with } \cosh(\lambda) = \exp(\Delta\tau U/2)$$

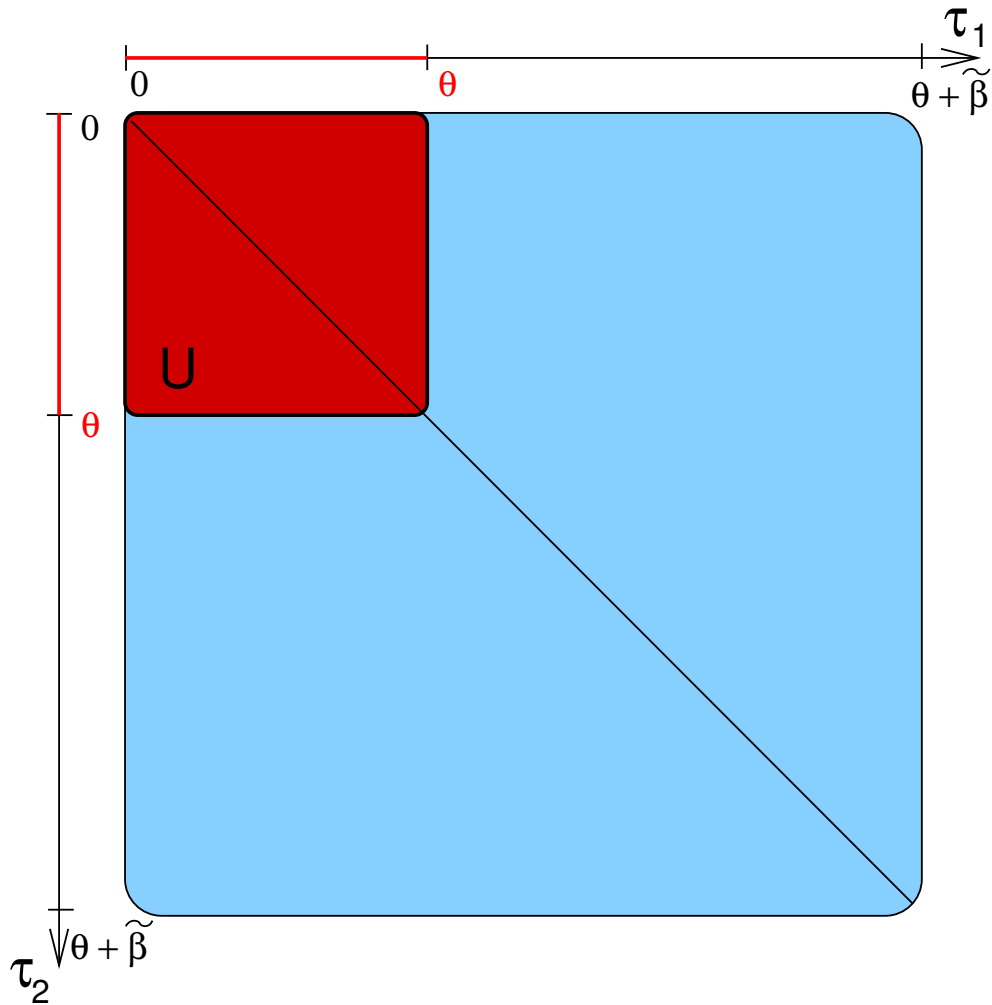
$\Rightarrow$  sum of non-interacting problems (MC sampling)

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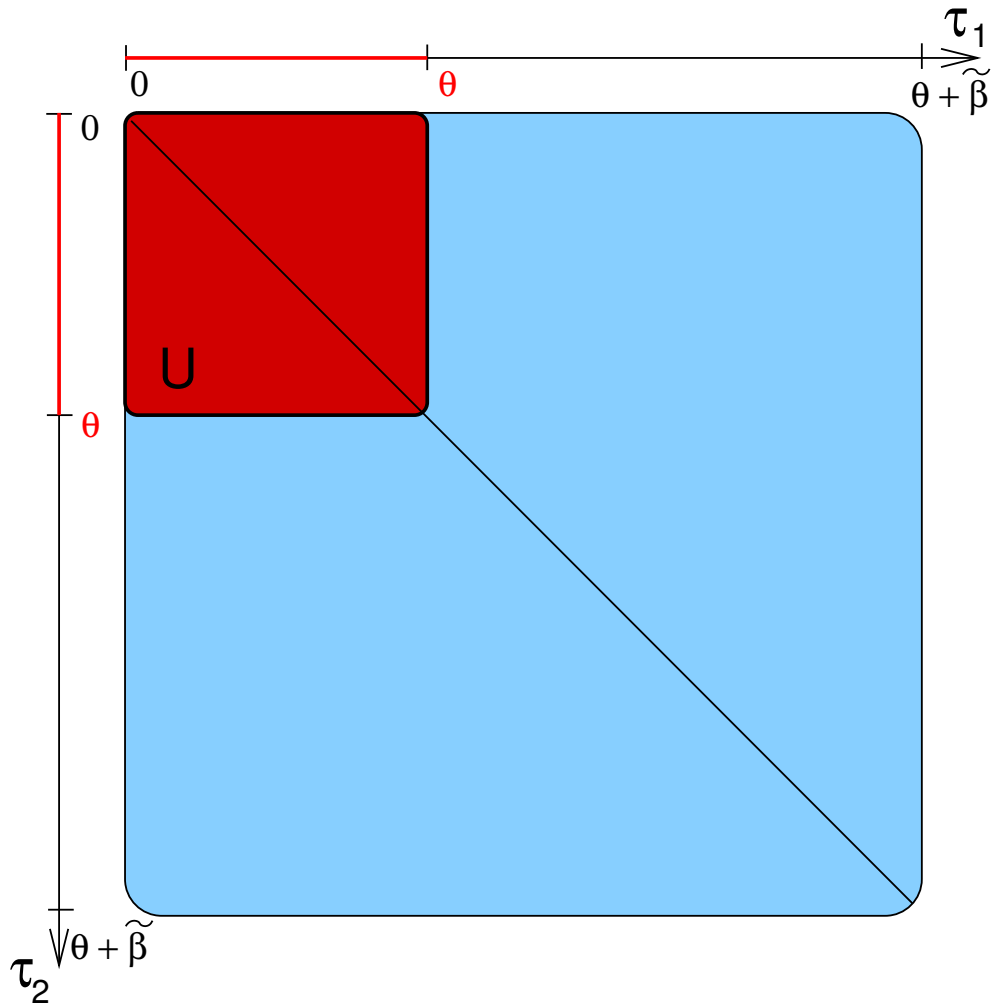
interaction **U** only in red part

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**Hubbard-Stratonovich** transformation

⇒ only Ising fields in red part

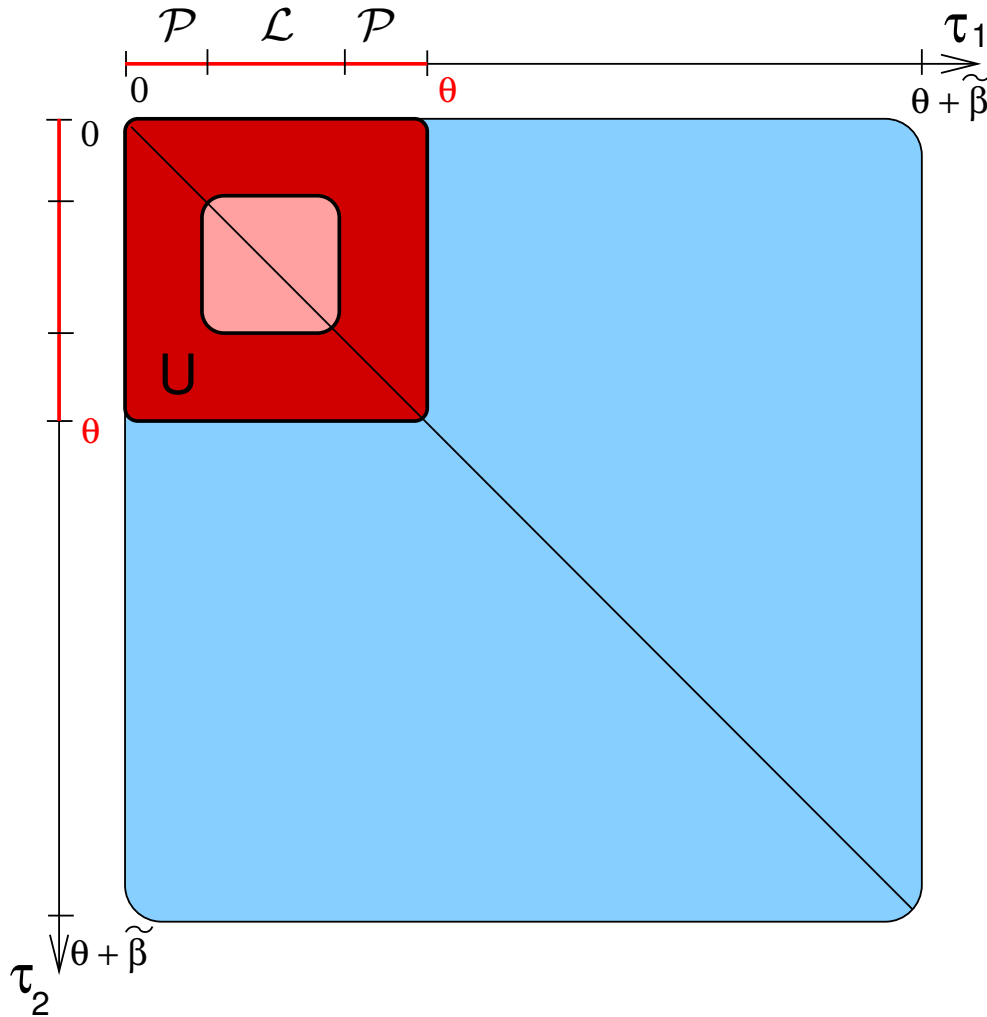
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Projection onto ground state needed

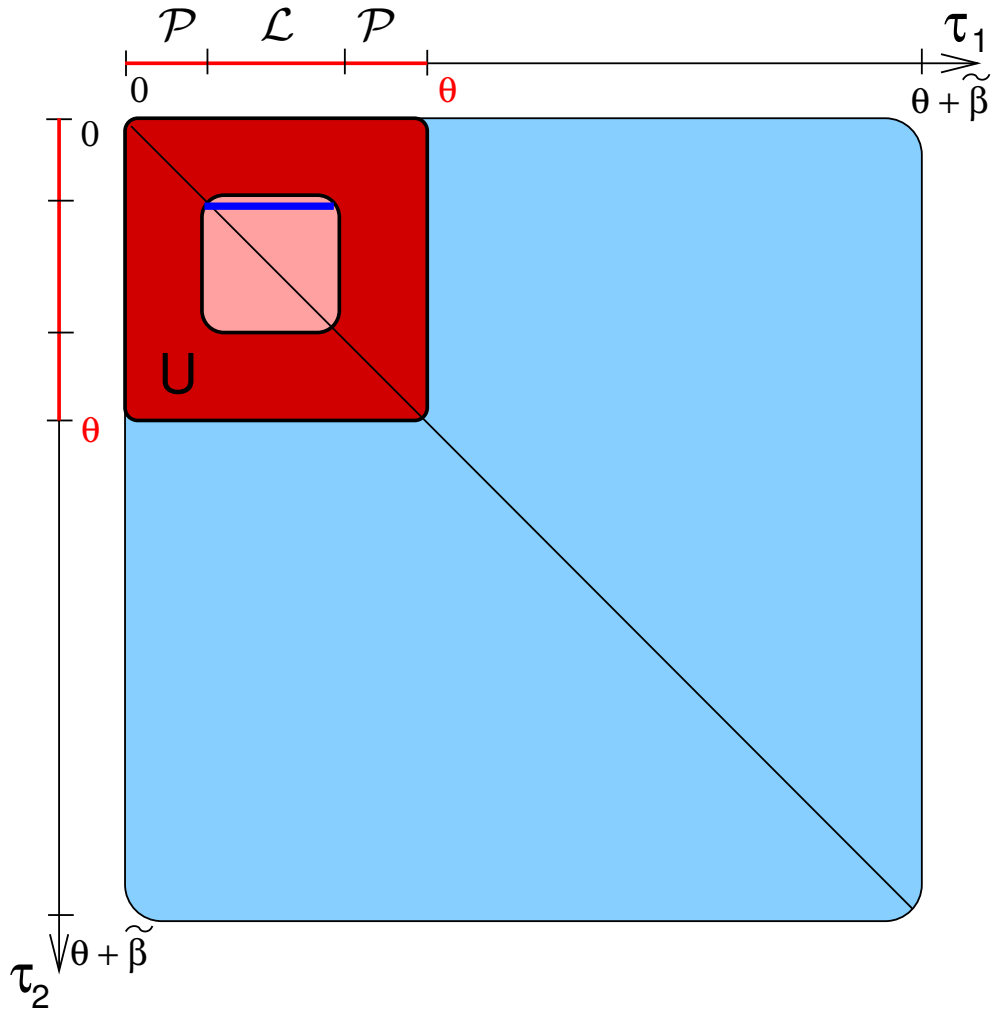
$\Rightarrow$  information only in **light red part** valid

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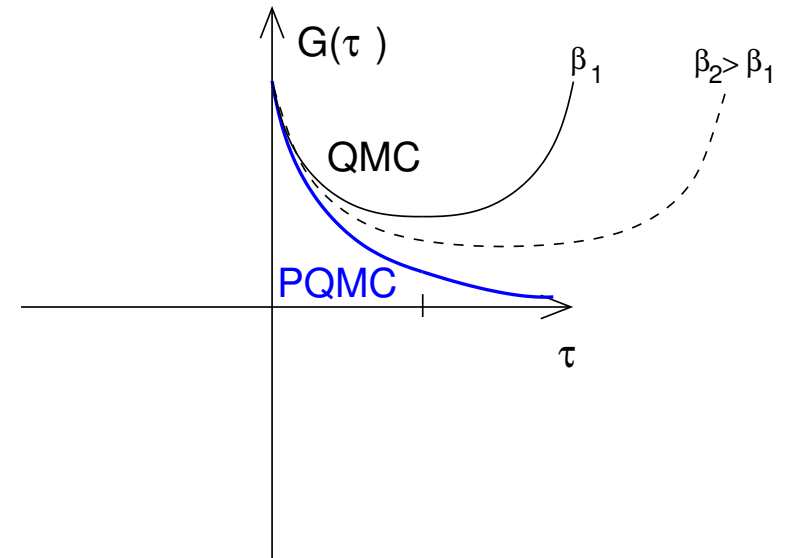
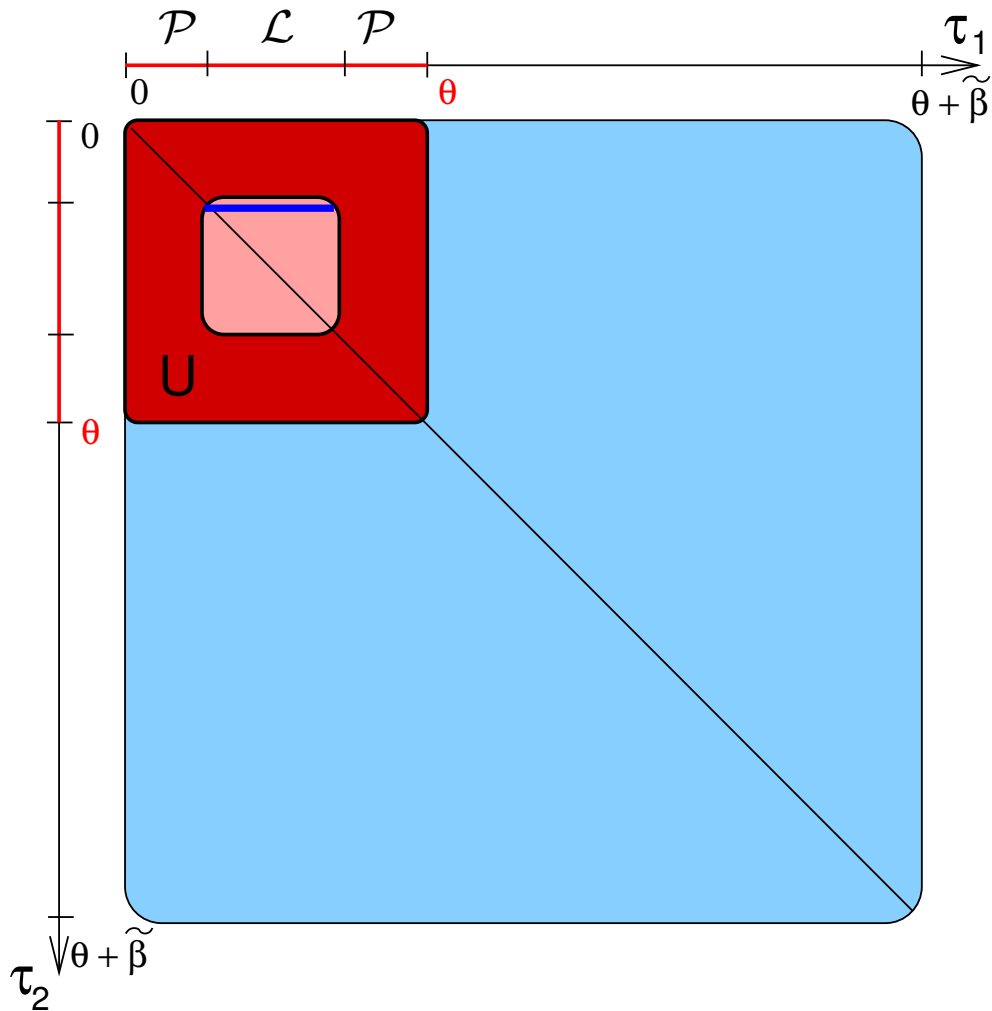


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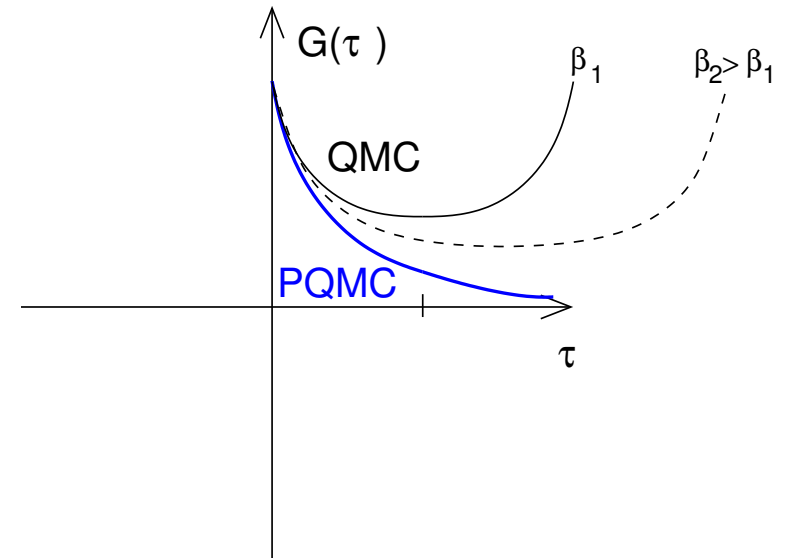
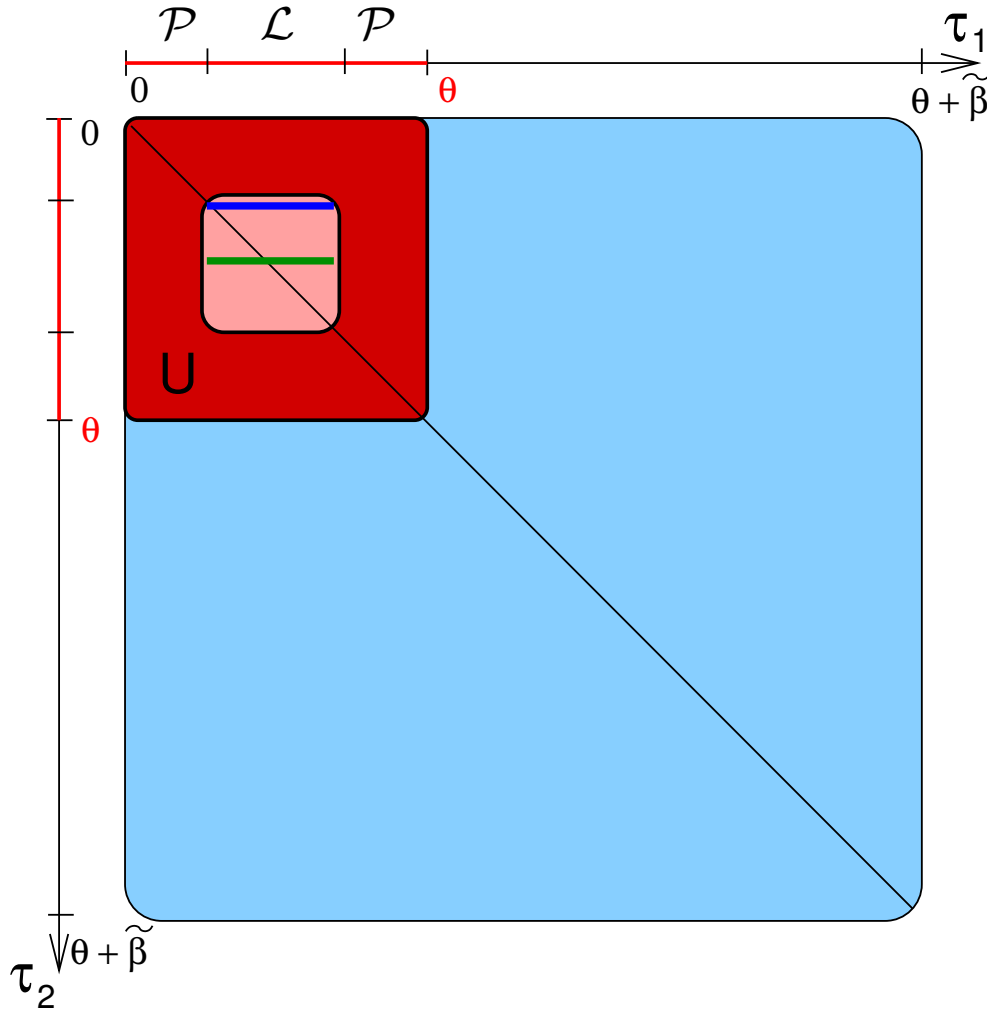


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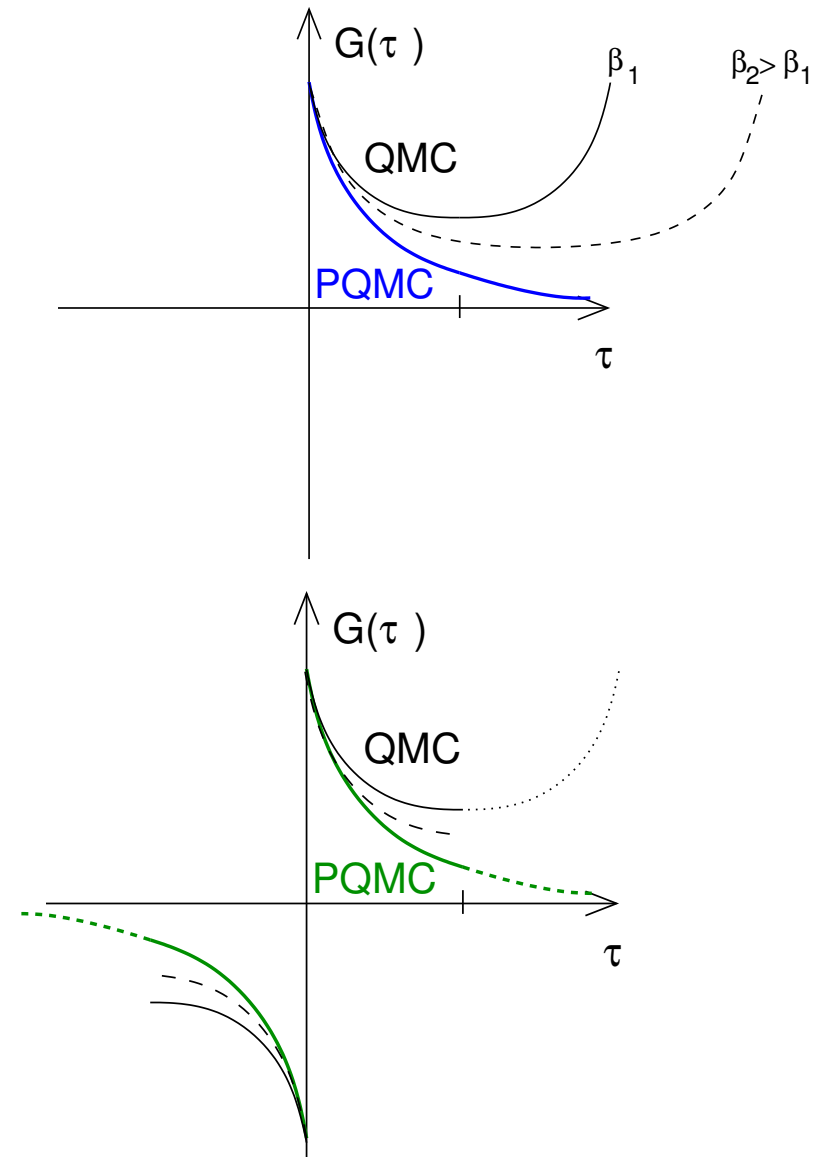
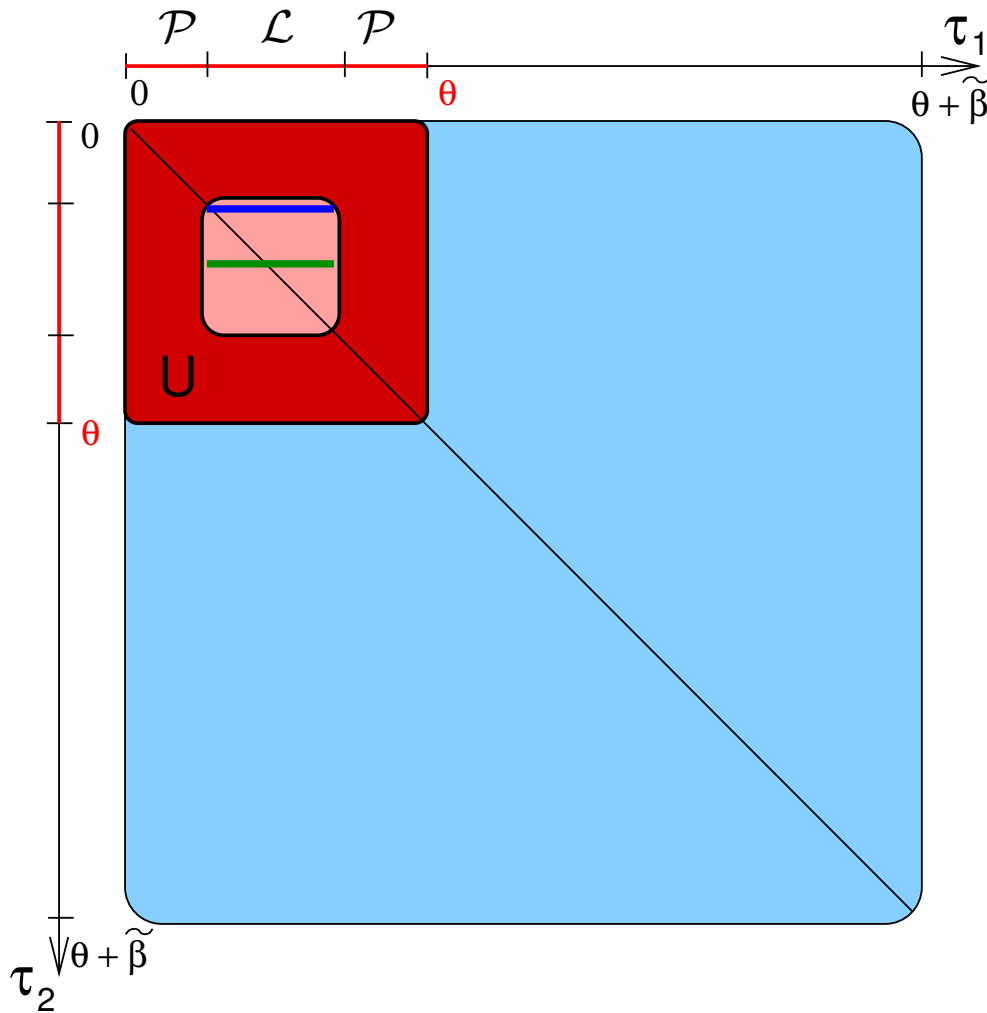


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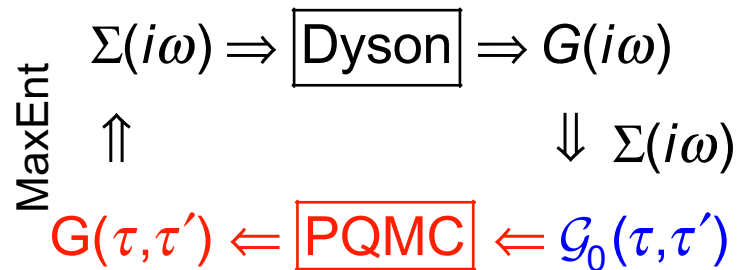


# DMFT(PQMC)

Feldbacher, Held, Assaad PRL'04

Ground state (PQMC):

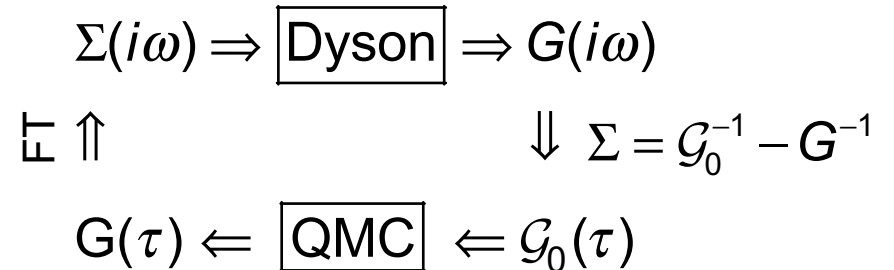
$$\mathcal{G}_0(\tau, \tau') : \tau, \tau' < \theta$$



finite- $T$  (HF-QMC):

$$\mathcal{G}_0^\sigma(\tau - \tau')$$

$$G = \sum_k \frac{1}{\omega - \varepsilon_k - \Sigma}$$



Maximum Entropy:

$$G(\tau) = \frac{1}{\pi} \int_0^\infty A(\omega) e^{-\omega\tau} d\omega$$

$$G(i\omega_n) = \frac{1}{\pi} \int_{-\infty}^\infty \frac{A(\omega)}{i\omega_n - \omega} d\omega$$

## II Results

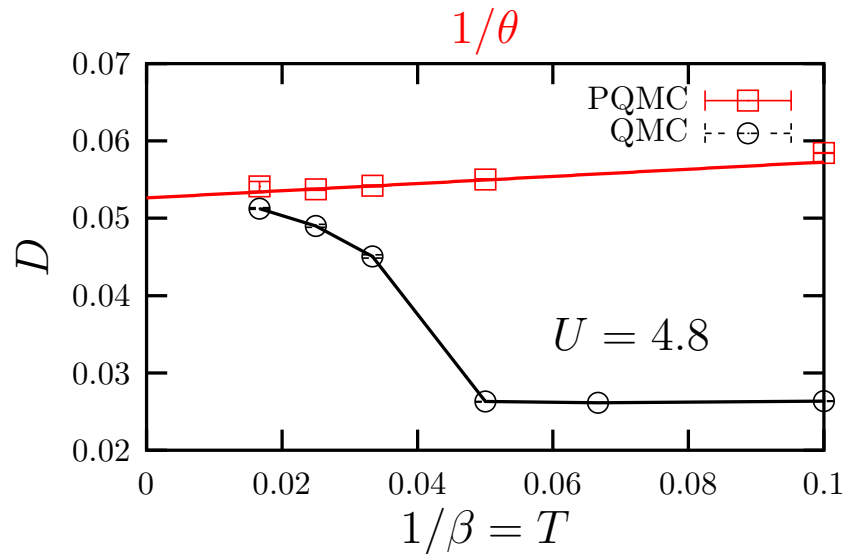
# One-band Hubbard model (half-filling)

Hamiltonian:

$$H = -t \sum_{\langle i,j \rangle \sigma} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + U \sum_{i\sigma} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$$

Bethe-DOS; bandwidth  $W = 4$

DMFT(QMC) vs. DMFT(PQMC)



# One-band Hubbard model (half-filling)

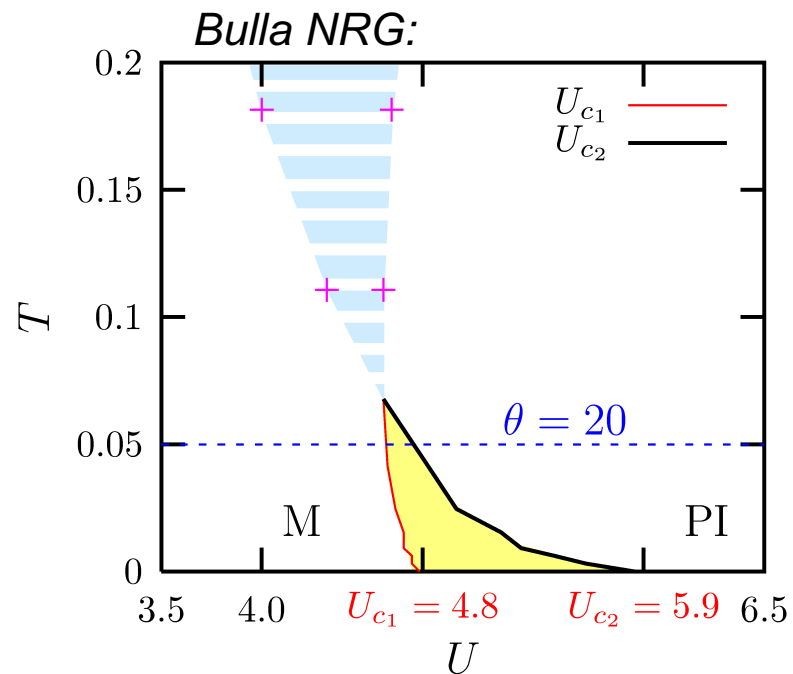
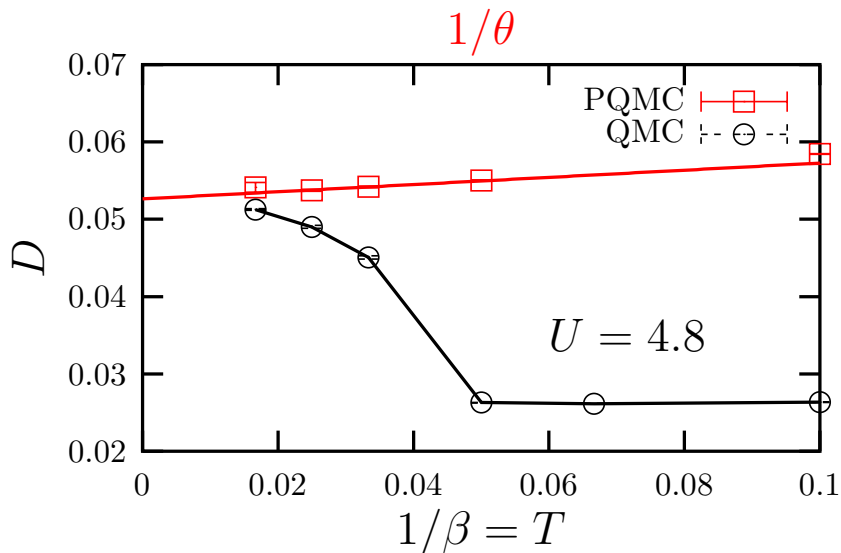
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Phase diagram DMFT(NRG)

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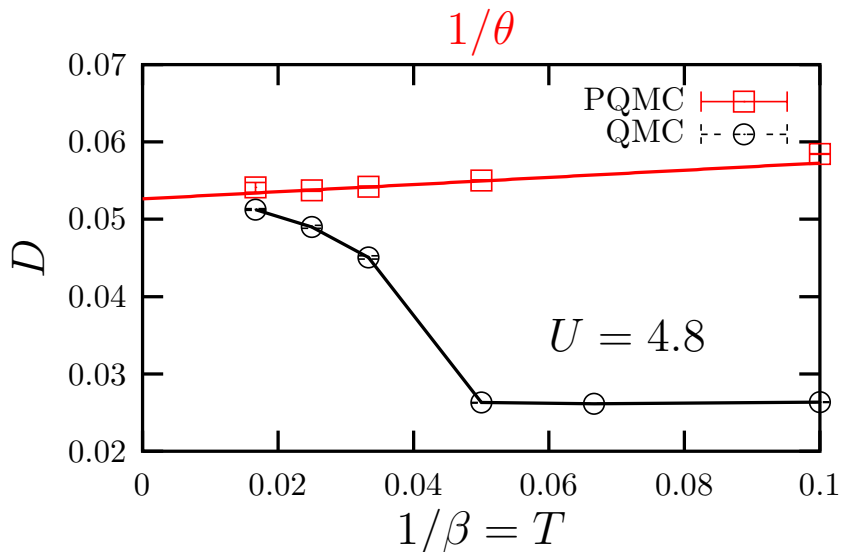
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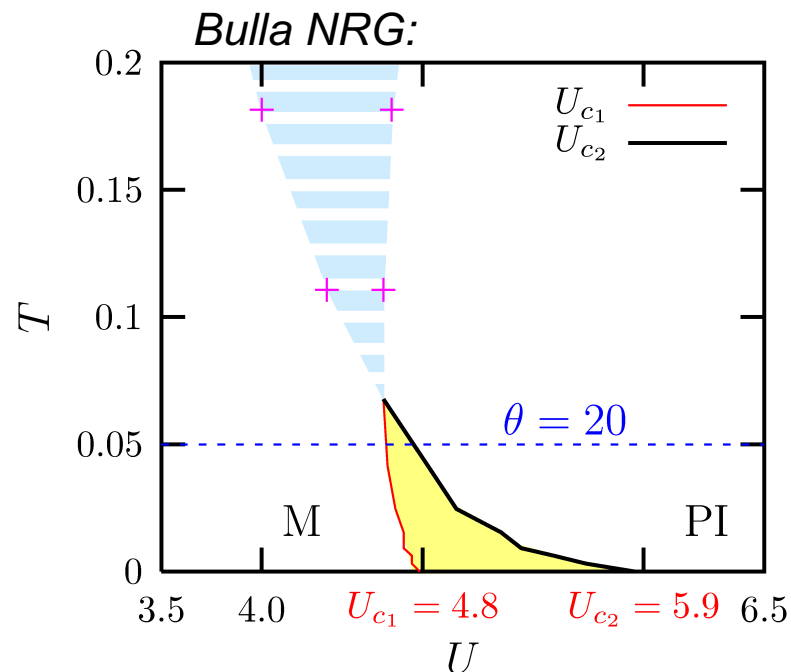
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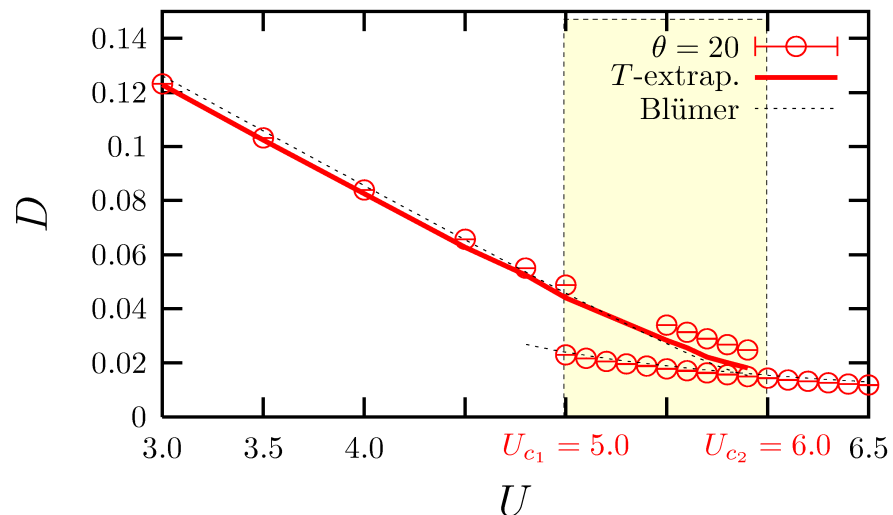


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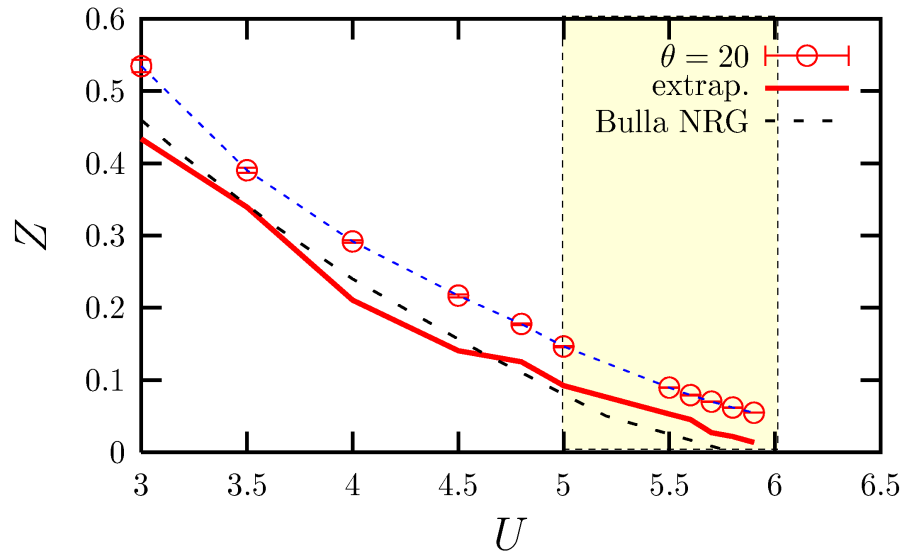


DMFT(PQMC) coexistence region

Feldbacher, Held, Assaad'04



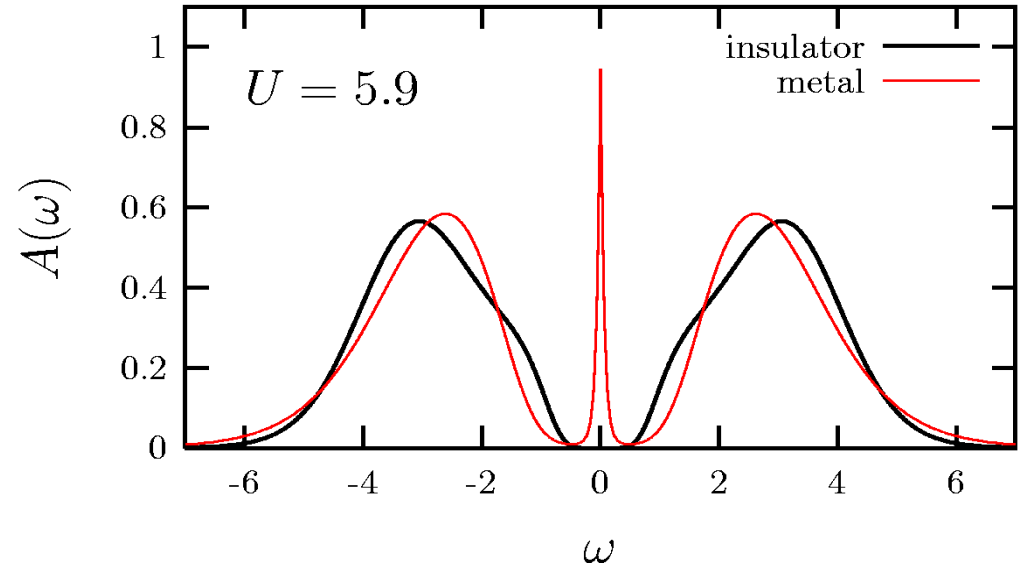
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$$Z = \left( 1 - \frac{\text{Im}\Sigma(i\omega_{n=1})}{\omega_{n=1}} \right)^{-1}$$

# Spectra

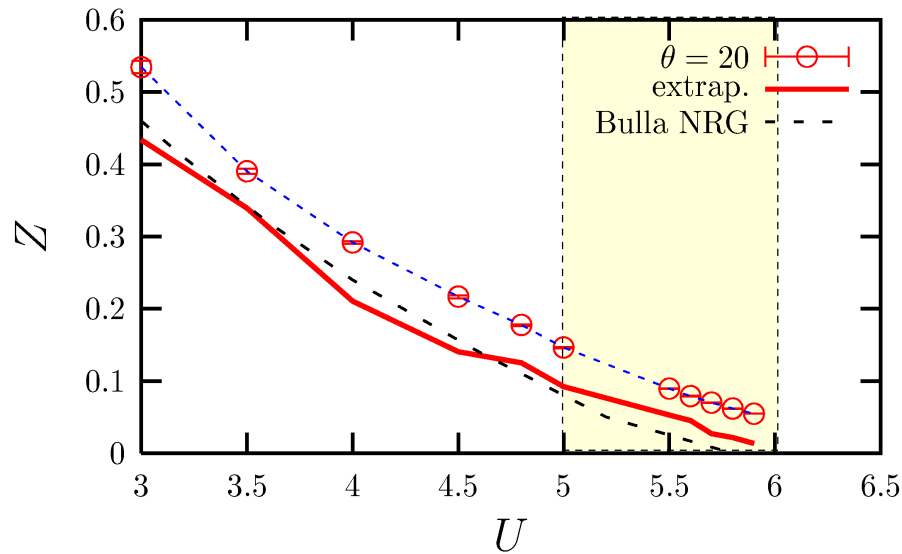
Feldbacher, Held, Assaad PRL'04;



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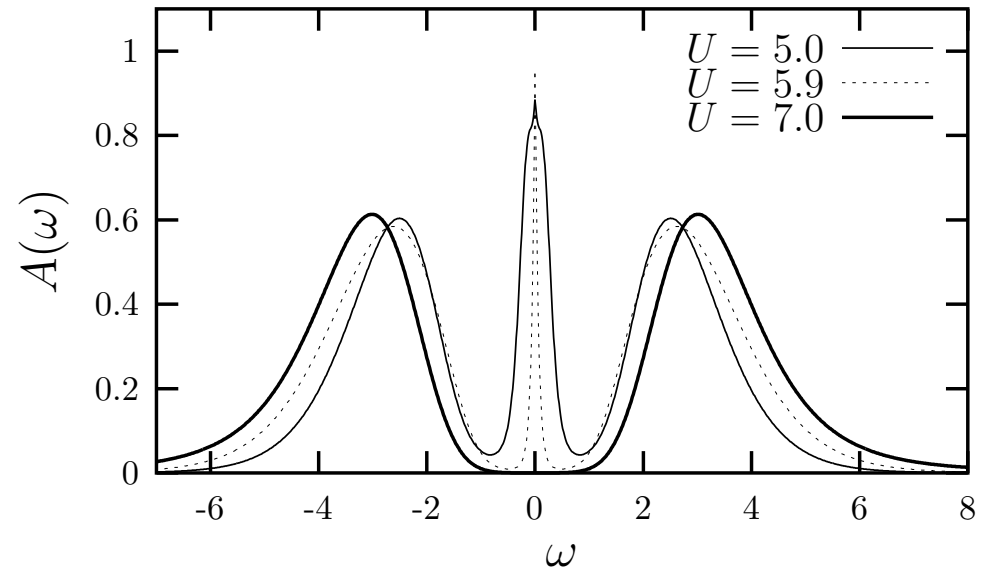
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# Spectra

Feldbacher, Held, Assaad PRL'04; PHYSICA B'05



Maximum Entropy:

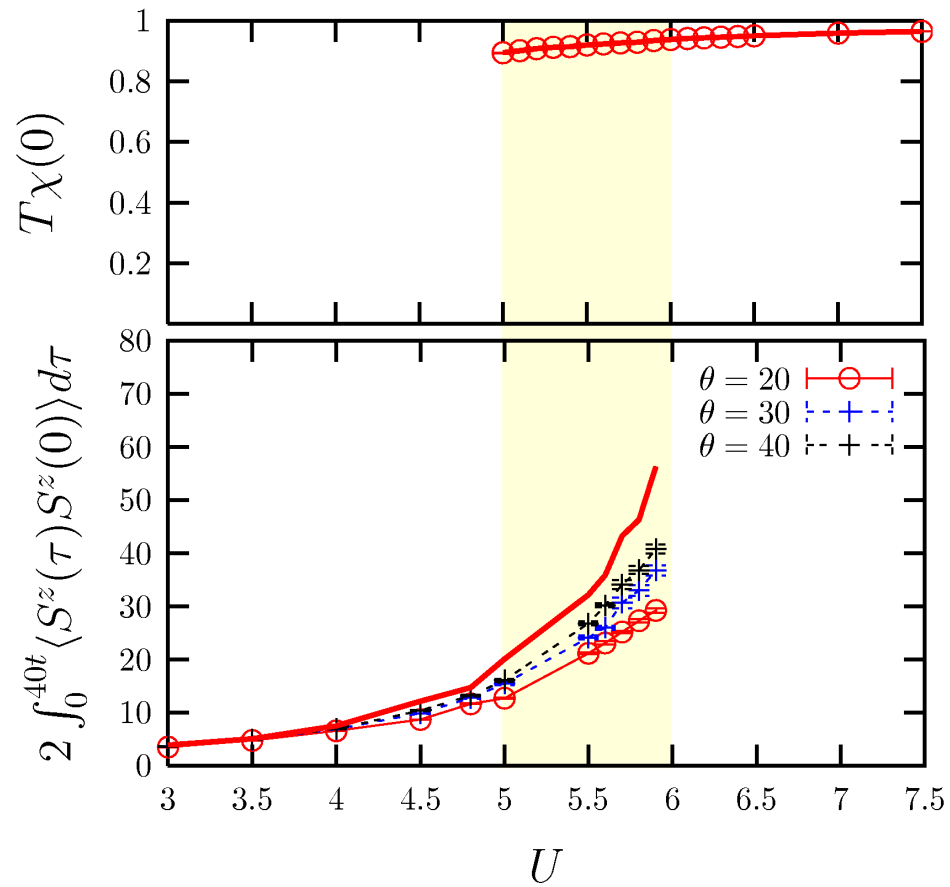
$$G(\tau) = \frac{1}{\pi} \int_0^{\infty} A(\omega) e^{-\omega\tau} d\omega$$

# Local spin susceptibility

Feldbacher, Held, Assaad PRL'04

Insulator:

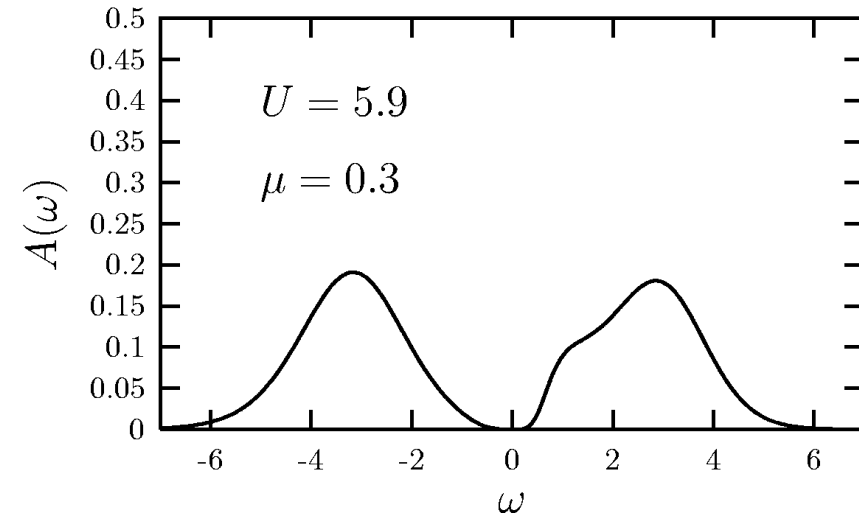
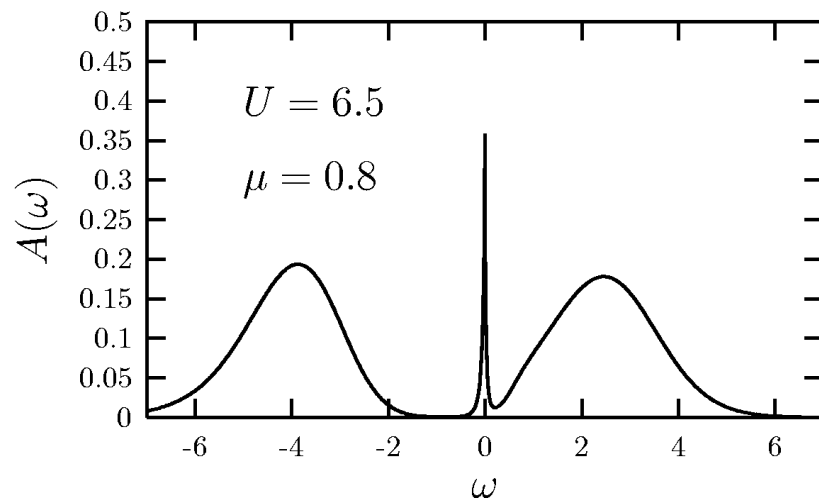
Metal:



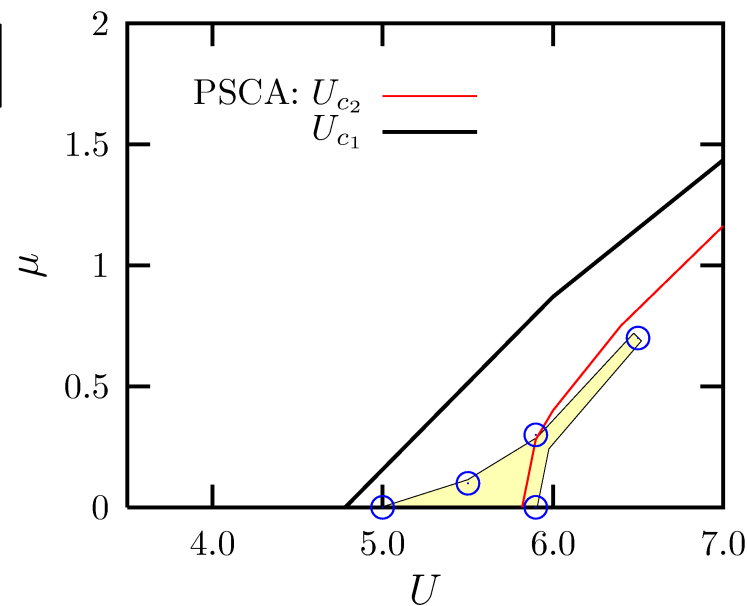
$$\int_0^{\tau_c} \langle S^z(\tau) S^z(0) \rangle d\tau$$

# Away from half-filling ...

Feldbacher, Held'04/05



Coexistence:



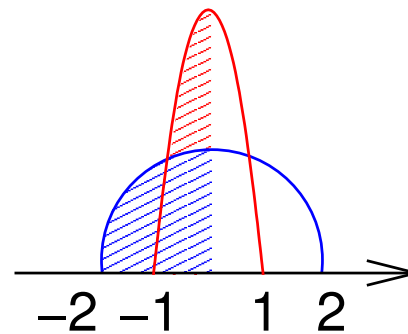
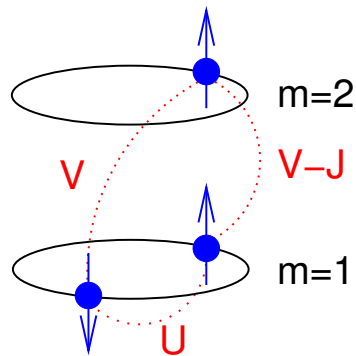
○ DMFT(PQMC)

— Fischer, Kotliar, Moeller'95

# Two-band Hubbard model (half-filling)

Arita, Held cond-mat/0504040

$$\begin{aligned}
 H = & - \sum_{\langle i,j \rangle m\sigma} t_m \hat{c}_{im\sigma}^\dagger \hat{c}_{jm\sigma} + U \sum_{im\sigma} \hat{n}_{im\uparrow} \hat{n}_{im\downarrow} + \sum_{i;(m,\sigma) < (m',\sigma')} (V - \delta_{\sigma\sigma'} J) \hat{n}_{im\sigma} \hat{n}_{im'\sigma'} \\
 & - \frac{1}{2} J \sum_{i\sigma;l \neq m} \hat{c}_{il\sigma}^\dagger \hat{c}_{il\bar{\sigma}} \hat{c}_{im\bar{\sigma}}^\dagger \hat{c}_{im\sigma} - \frac{1}{2} J \sum_{i\sigma;l \neq m} \hat{c}_{il\sigma}^\dagger \hat{c}_{il\bar{\sigma}}^\dagger \hat{c}_{im\sigma} \hat{c}_{im\bar{\sigma}}
 \end{aligned}$$



## Controversy:

**One** first-order Mott-Hubbard transition

[Liebsch'03'04; DMFT(QMC) only **first line** of  $H$ ]

or **two** [Koga, Kawakami, Rice, Anisimov'04; DMFT(ED)]?

We:

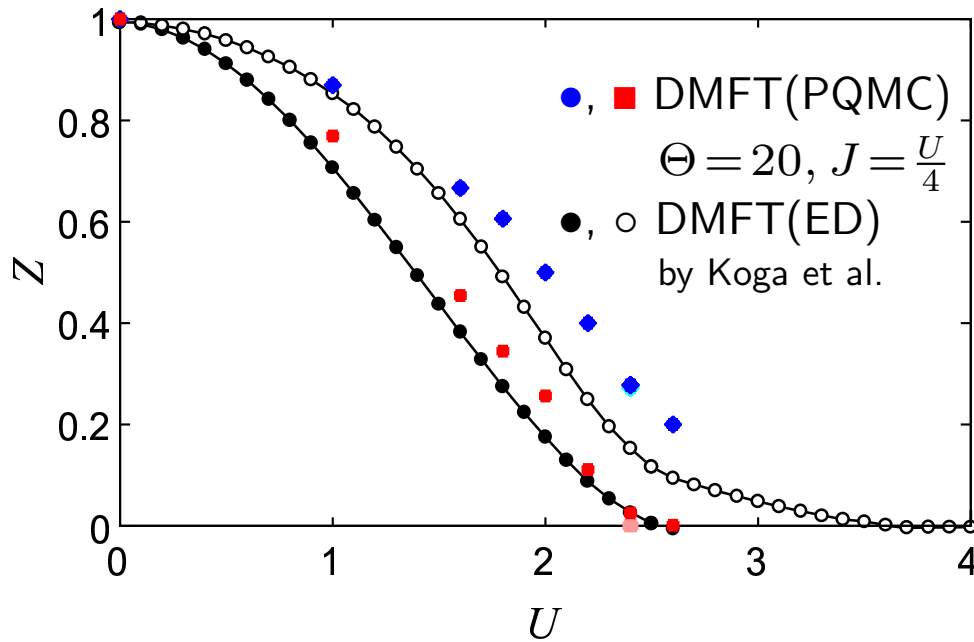
**DMFT(PQMC)**

**Correct multiplet structure** via

Hubbard-Stratonovich decoupling of Sakai, Arita, Aoki'04

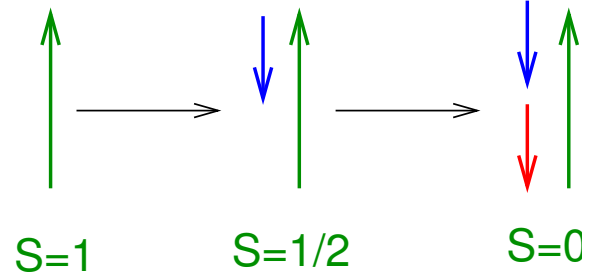
# DMFT(PQMC) results

Arita, Held cond-mat/0504040



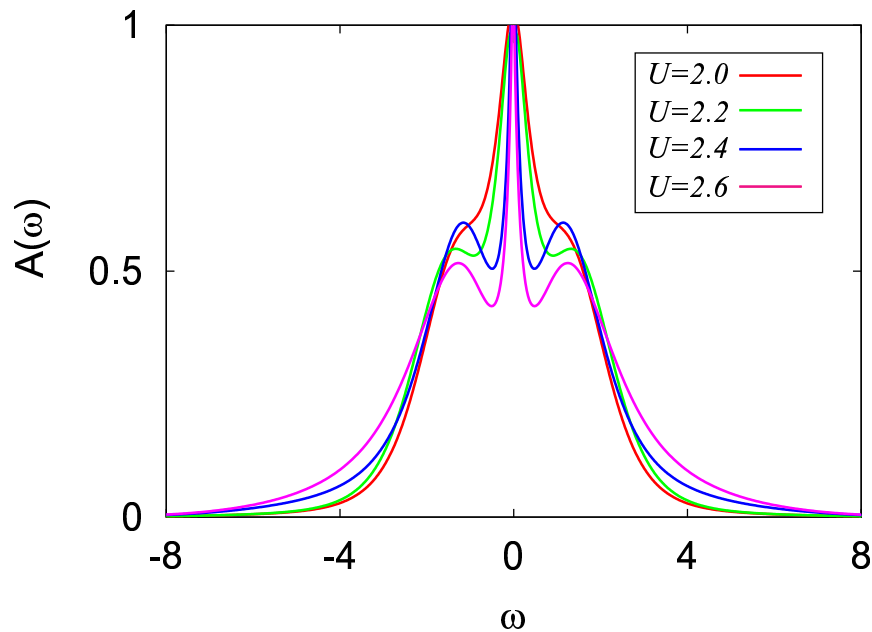
two consecutive 1. order transitions!

$J >$  effective  $T_K$ :

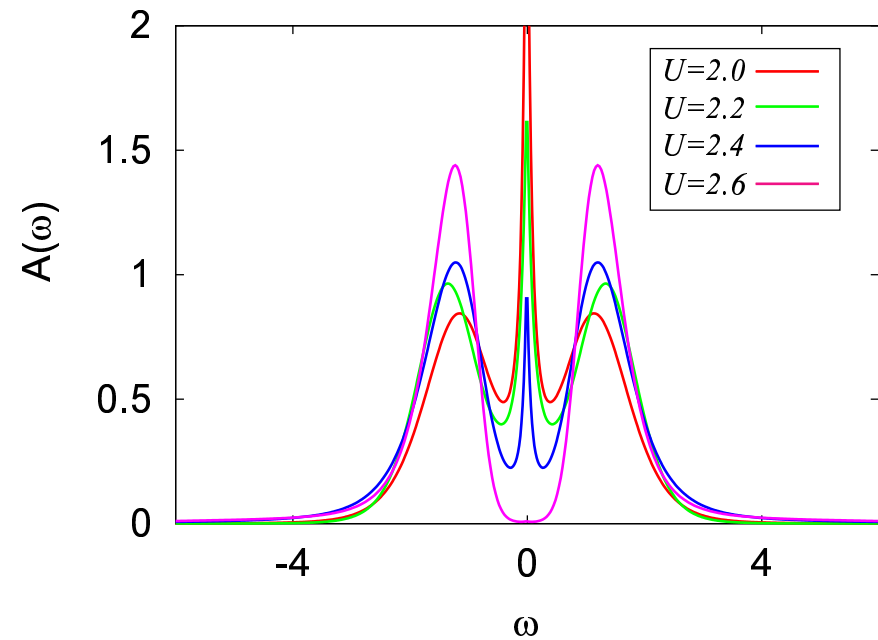


two stage screening

wide band



narrow band



# DCA(PQMC) study of the $t$ - $t'$ Hubbard model

Arita, Held'05

$$H = -t \sum_{\langle i,j \rangle_{\text{NN}} \sigma} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + t' \sum_{\langle i,j \rangle_{\text{NNN}} \sigma} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + U \sum_{i\sigma} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$$

Does the Hubbard model describe **d-/p-wave** superconductivity??

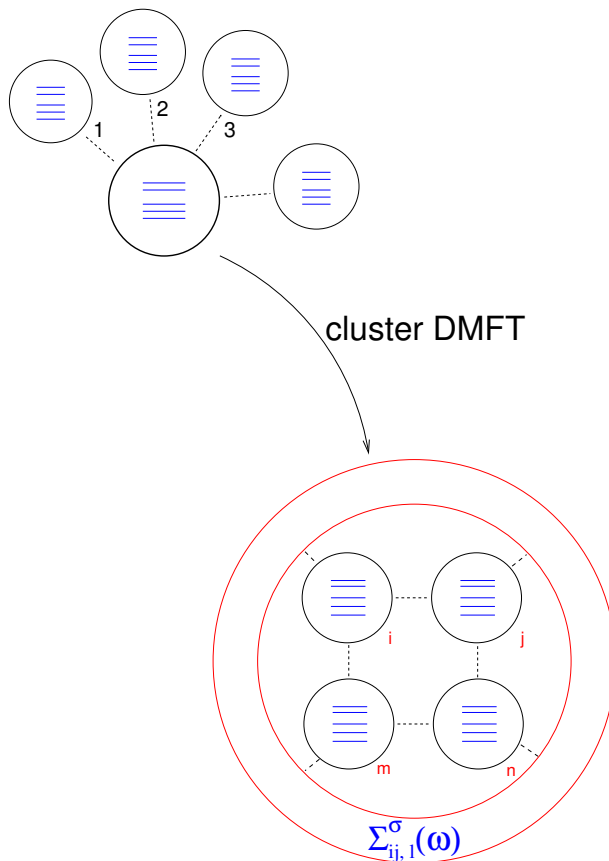
# DCA(PQMC) study of the $t$ - $t'$ Hubbard model

Arita, Held'05

$$H = -t \sum_{\langle i,j \rangle_{\text{NN}} \sigma} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + t' \sum_{\langle i,j \rangle_{\text{NNN}} \sigma} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + U \sum_{i\sigma} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$$

Does the Hubbard model describe **d-/p-wave** superconductivity??

k-dependence essential  $\rightarrow$  DCA



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Arita, Held'05

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Does the Hubbard model describe **d-/p-wave** superconductivity??

Some Indications for superconductivity:

**fRG** Honerkamp, Salmhofer'01; Katanin, Kampf'03

→ weak coupling

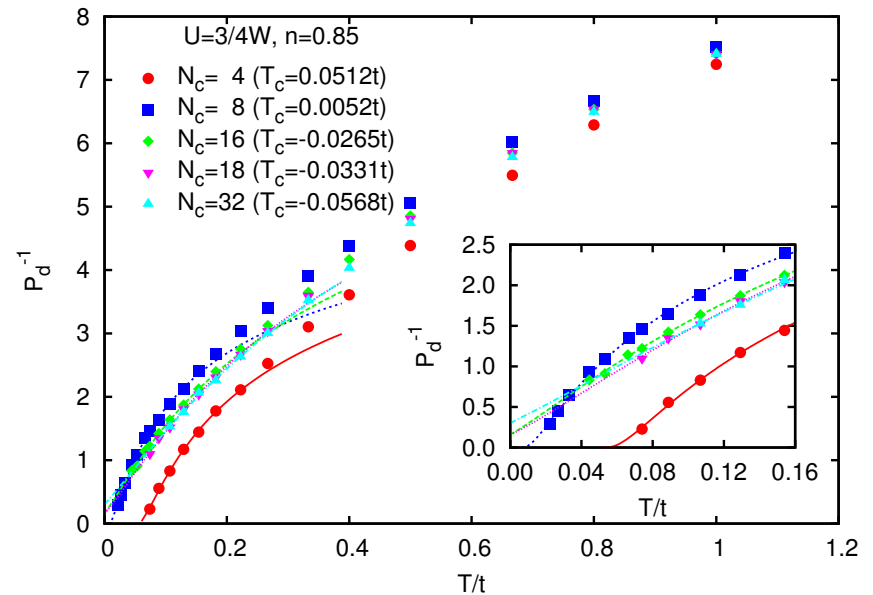
**DCA(QMC)** Hettler *et al.*'98; Lichtenstein, Katsnelson'01;

Maier *et al.* '04, '05...

→ definite answer difficult

$T \rightarrow 0$  extrapolation

$N_c \rightarrow \infty$  extrapolation

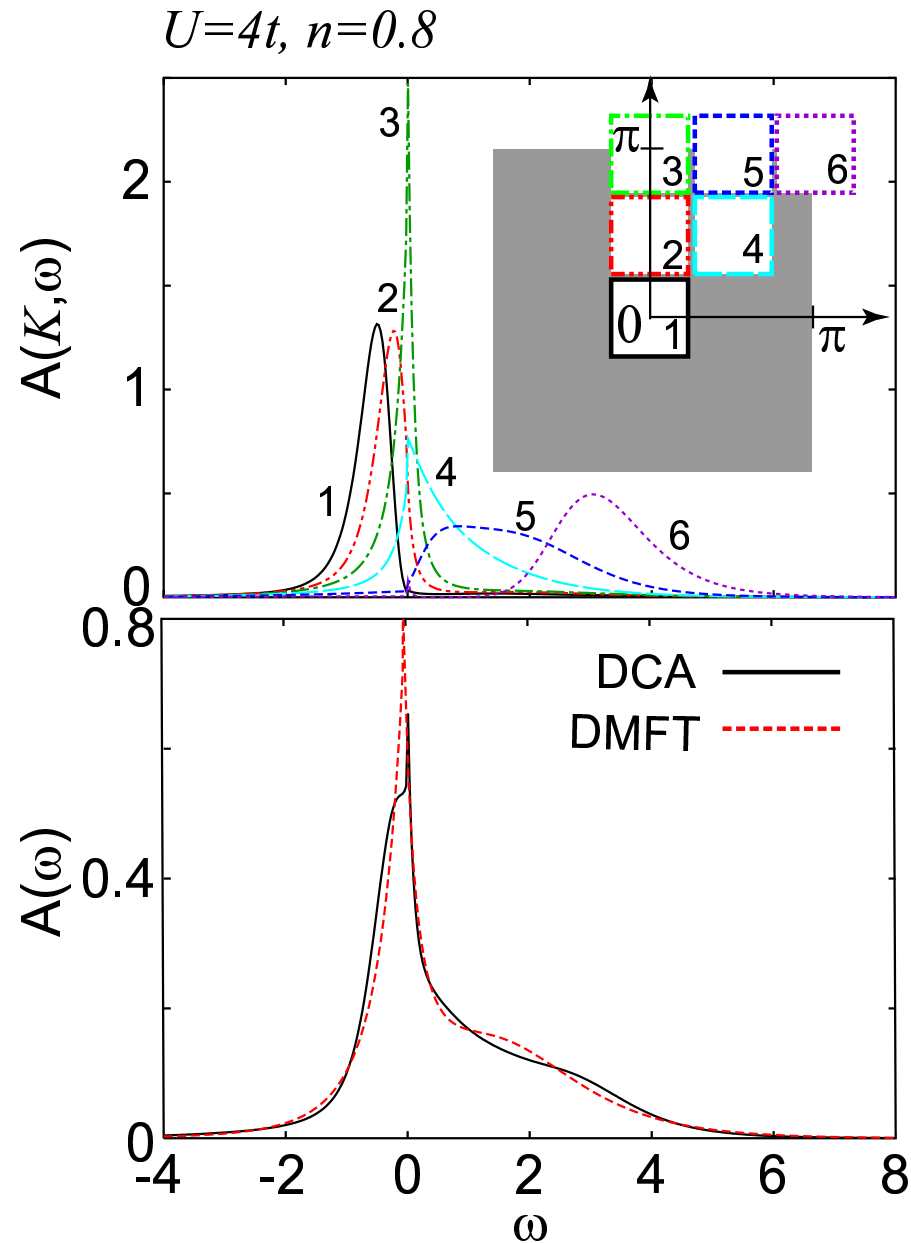


Maier *et al.* cond-mat/0409669

We: **DCA(PQMC)** → fast, direct route to  $T=0$

# Spectral function within PM phase

Arita, Held'05

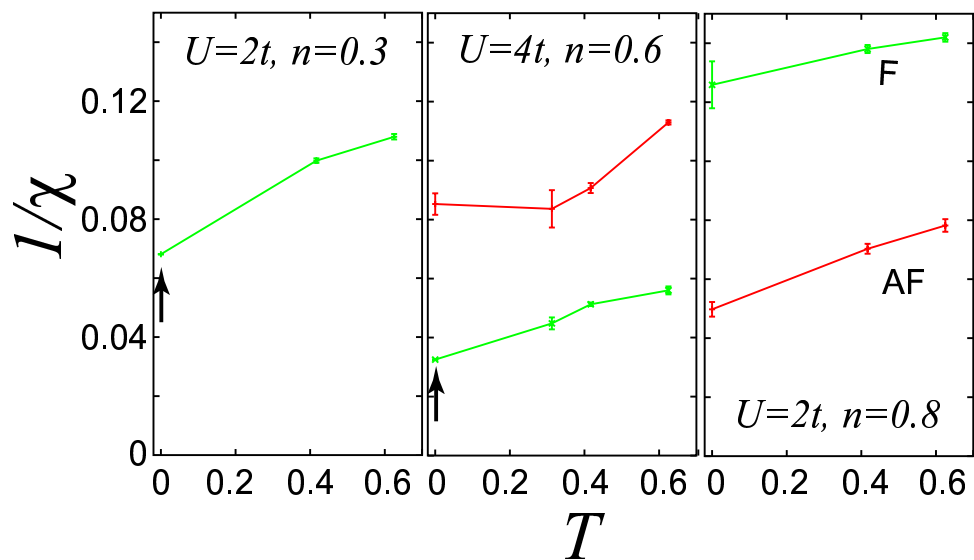


Here and in the following:  $t' = 0.4t, N_c = 4 \times 4 = 16$

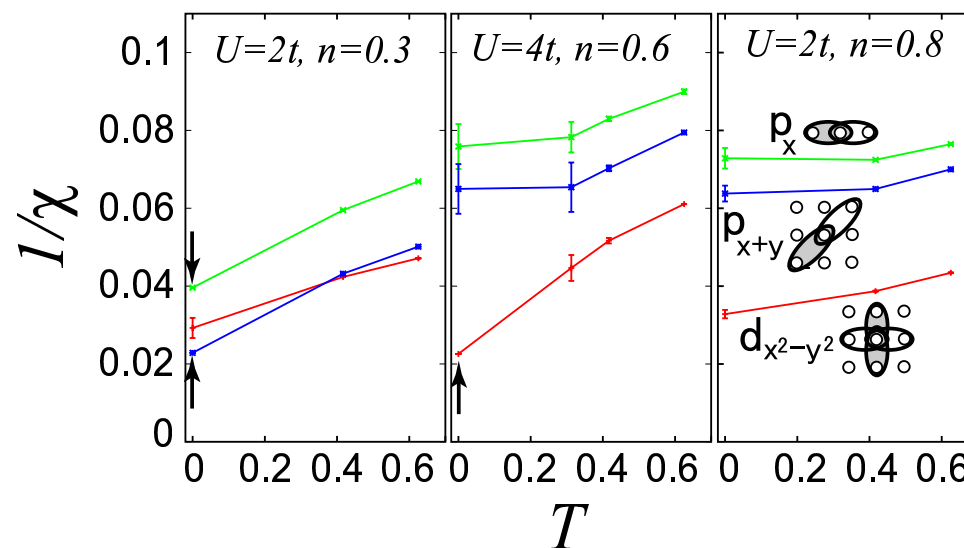
# Cluster susceptibilities

Arita, Held'05

Magnetic susceptibility:

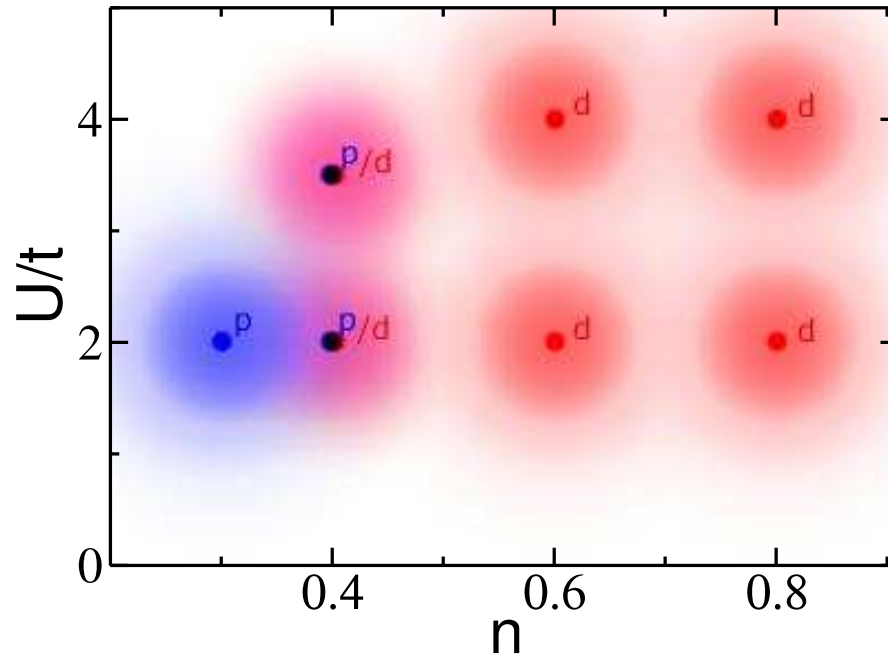


Pair susceptibility:



# Diagram of dominant pairing symmetry

Arita, Held'05



# Conclusion

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  - works with  $G_{T=0}(\tau)$  instead of  $G_T(\tau)$
  - easy to implement
- Applications so far:
  - One-band Hubbard model at and away from half-filling
  - Two-band Hubbard model — two consecutive Mott-Hubbard transitions
  - DCA(PQMC) — pair susceptibility in the  $t$ - $t'$  Hubbard model
- Outlook:  
PQMC for LDA+DMFT and LDA+DCA