Current-Induced Domain-Wall Dynamics in Ferromagnetic Nanowires

Benjamin Krüger

17.11.2006
## Model
- The Micromagnetic Model
- Current Induced Magnetisation Dynamics
- Phenomenological Description
- Experimental Setup

## Analytic Calculations
- Equations of Motion
- Domain Wall Quasiparticle
- Solution of the Wall Mode
- Phenomenological Explanation
- Geometric Corrections

## Numeric Calculations
- The Eigenmodes
- Comparing the Model to the Numerical Results

## Comparison with Experiment
In the micromagnetic model we assume the magnetisation at each point as a continuous vector.

The time evolution of our system can be described by the Landau-Lifshitz-Gilbert-Equation.

\[
\frac{d\vec{M}}{dt} = -\gamma' \vec{M} \times \vec{H}_{\text{eff}} - \gamma' \alpha \vec{M} \times (\vec{M} \times \vec{H}_{\text{eff}})
\]

The first term can be proven by quantum mechanic and is correct to first order in \(\hbar\).

The second part is a phenomenological damping term.
The magnetisation dynamics will be described by the extended Landau-Lifshitz-Gilbert-Equation from Zhang and Li\textsuperscript{1}.

\[
\frac{d\vec{M}}{dt} = \underbrace{-\gamma' \vec{M} \times \vec{H}_{\text{eff}}} - \underbrace{-\frac{\alpha \gamma'}{M_s} \vec{M} \times (\vec{M} \times \vec{H}_{\text{eff}})} \quad \text{precession term} \\
- \underbrace{b_j' (1 + \alpha \xi) \vec{M} \times (\vec{M} \times (\vec{j} \cdot \vec{\nabla}) \vec{M})} \quad \text{damping term} \\
- \underbrace{\frac{b_j'}{M_s^2} (\xi - \alpha) \vec{M} \times (\vec{j} \cdot \vec{\nabla}) \vec{M}} \quad \text{motion term} \\
- \underbrace{\frac{b_j'}{M_s} (\xi - \alpha) \vec{M} \times (\vec{j} \cdot \vec{\nabla}) \vec{M}} \quad \text{distortion term}
\]

Phenomenological Description

\[-\gamma' \vec{M} \times \vec{H}_{\text{eff}} - \frac{\alpha \gamma'}{M_s} \vec{M} \times (\vec{M} \times \vec{H}_{\text{eff}})\]

precession term \hspace{2cm} damping term
Phenomenological Description

\[-\frac{b'_j}{M_s^2}(1 + \alpha \xi) \vec{M} \times \left( \vec{M} \times (\vec{j} \cdot \vec{\nabla}) \vec{M} \right)\]

motion term

In the case of homogeneous current density \( \vec{j}(x) = \vec{j}_0 \)

\[
\frac{d\vec{M}}{dt} = b'_j(1 + \alpha \xi)(\vec{j} \cdot \vec{\nabla})\vec{M} = -(\vec{v} \cdot \vec{\nabla})\vec{M}
\]

The solution is a magnetisation of the form \( \vec{M}(\vec{r} - \vec{v}(t - t_0), t_0) \)
\[- \frac{b_j'}{M_s} (\xi - \alpha) \vec{M} \times (\vec{j} \cdot \nabla) \vec{M} \]

\[\text{distortion term}\]
A semicircle permalloy nanowire is placed in an external magnetic field.

The system is prepared by applying a large magnetic field in y-direction. Reducing the magnetic field we end up with a domain wall at the bottom of the wire.

The type of the wall depends on the width \( w \) and the thickness \( t \) of the wire\(^2\).

Here we will assume a small cross section $w = 10\text{nm}$, $t = 10\text{nm}$ and $S = wt = 100\text{nm}^2$ to get a Néel wall.

The wall can be excited by an oscillating current which is applied at the two contacts.

The contacts are arranged in an angle of 45 degree to have enough distance to the domain wall as well as to the ends of the wire.
We will start with
- an equation of motion in a straight wire
- an arbitrary Zeeman field in $x$ direction which depends on the position $X$ of the domain wall

Later on we will specify the equation of motion to the sine shaped field of a semicircle wire.

The magnetisation is expressed in spherical coordinates via the polar angle $\theta$ and the azimuthal angle $\phi$. 
In the absence of electric current and external magnetic field the energy of the domain-wall for $\phi = 0$ is given by

$$E = S \int \left( A \left( \frac{\partial \theta(x)}{\partial x} \right)^2 + K \sin^2(\theta(x)) \right) dx,$$

This functional can be minimised by a Néel wall which is given by the angle

$$\theta = 2 \arctan e^{\frac{x-X}{\lambda}}$$
Following the description of Schryer and Walker\textsuperscript{3} for the movement in a magnetic field we introduce two dynamical variables the centre of the wall $X$ and the angle around the wire axis $\phi(x) = \phi$ that is uniform along the wire.

\textsuperscript{3} J. Appl. Phys. 45, 5406 (1974)
With the Zhang and Li equation, the static solution of the wall, and the energy density

\[ W = A \left( \frac{\partial \theta}{\partial x} \right)^2 + K \sin^2(\theta) + K_{\perp} \sin^2(\theta) \sin^2(\phi) \]

\[ \text{exchange} \quad \text{anisotropy} \quad \text{perpendicular anisotropy} \]

\[ - M_s H_{\text{ext}} \cos(\theta) \mu_0. \]

Zeeman

the linearised equations of motion for the domain wall yield

\[ \dot{X} = - \frac{2\gamma' K_{\perp}}{\mu_0 M_s} \phi - \lambda \gamma' \alpha H_{\text{ext}} - b'_j(1 + \alpha \xi) j \]

and

\[ \dot{\phi} = - \frac{2\gamma' \alpha K_{\perp}}{\mu_0 M_s} \phi + \gamma' H_{\text{ext}} + \frac{b'_j (\xi - \alpha)}{\lambda} j. \]
The linearised equations of motion for the domain wall yield

\[
\dot{X} = -\frac{2\gamma' K_\perp}{\mu_0 M_s} \phi - \lambda \gamma' \alpha H_{\text{ext}} - b'_j(1 + \alpha \xi)j
\]

and

\[
\dot{\phi} = -\frac{2\gamma' \alpha K_\perp}{\mu_0 M_s} \phi + \gamma' H_{\text{ext}} + \frac{b'_j(\xi - \alpha)}{\lambda} j.
\]

The domain wall moves as one particle with

- the position \(X\)
- the conjugated momentum \(\phi\)

This give rise to the assumption that

- a mass can be assigned to the particle
- a force on the particle can be introduced
The domain wall mass can be obtained by calculating $\phi$ in absence of electric current and external field

$$\phi = -\dot{X} \frac{\mu_0 M_s}{2\lambda \gamma' K_\perp}$$

This can be inserted in the energy density

$$\frac{1}{2} m \dot{X}^2 = E$$

$$= S \int dx \ K_\perp \sin^2(\theta) \left( \dot{X} \frac{\mu_0 M_s}{2\lambda \gamma' K_\perp} \right)^2$$

$$= \frac{1}{2} S \mu_0^2 M_s^2 \frac{1}{2 \lambda \gamma'^2 K_\perp} \dot{X}^2.$$ 

Comparing both sides leads to the value of the domain wall mass

$$m = \frac{S \mu_0^2 M_s^2}{\lambda \gamma'^2 K_\perp}.$$
The force on the wall is given by

$$\frac{F}{m} = \ddot{X} = -\frac{2\lambda \gamma' K_\perp}{\mu_0 M_s} \dot{\phi} - \lambda \gamma' \alpha \dot{H}_{\text{ext}} - b'_j (1 + \alpha \xi) \dot{j}.$$  

Inserting the equations of motion one ends up with the expression

$$\frac{F}{m} = -\frac{2\gamma' K_\perp}{\mu_0 M_s} \left( \lambda \gamma H + \xi b_j j + \alpha \dot{X} \right) - (1 + \alpha \xi) b'_j j - \gamma' \lambda \alpha \dot{H}.$$  

With currents and fields which are not time dependent the force is zero if

$$\dot{X} = -\frac{\lambda \gamma}{\alpha} H - \frac{\xi}{\alpha} b_j j.$$
Introducing the sine shaped external field of a curved wire \( (H_{\text{ext}} = \sin \left( \frac{X}{r} \right) ) \) we get for small \( X \)

\[
\begin{align*}
\dot{X} &= -\frac{\lambda \gamma' \alpha H_0}{r} X - \frac{2\lambda \gamma' K_{\perp}}{\mu_0 M_s} \phi - b'_j (1 + \alpha \xi) j \\
\dot{\phi} &= \frac{\gamma' H_0}{r} X - \frac{2\gamma' \alpha K_{\perp}}{\mu_0 M_s} \phi + \frac{b'_j (\xi - \alpha)}{\lambda} j.
\end{align*}
\]

This is an inhomogeneous system of explicit linear differential equations of first order.
The solution of the linear differential equations is

\[
\begin{pmatrix}
X(t) \\
\phi(t)
\end{pmatrix} = \frac{e^{i\Omega t}}{\omega_r^2 - \Omega^2 + i\Omega \Gamma} \vec{F}
\]

with

\[
\Gamma = \alpha \gamma' \left( \frac{\lambda H_0}{2r} + \frac{K_\perp}{\mu_0 M_s} \right)
\]

\[
\omega_r = \sqrt{\frac{2\gamma'^2 H_0 \lambda K_\perp}{\mu_0 M_s r}} (1 + \alpha^2)
\]

\[
\vec{F} = -bjj_0 \left( \frac{2\gamma' K_\perp \xi}{\mu_0 M_s} + \frac{1 + \alpha \xi}{1 + \alpha^2} \frac{i\Omega}{\lambda} \right) e^{i\Omega t}
\]

Plus an exponentially damped starting configuration.
The force per wall mass has three different sources

- the terms proportional to $\Omega$ are direct results of the spin torque
- the term proportional to $K_\perp$ is a result of the precession in the effective field $H_a$
- the term proportional to $H_\parallel$ is the precession in the external field

\[
\frac{\vec{F}}{m} = -b j_0 \left( \frac{2\gamma' K_\perp \xi}{\mu_0 M_s} + \frac{1+\alpha \xi}{1+\alpha^2} i\Omega \right) e^{i\Omega t}
\]
Due to the geometry of curved wire we have to take into account two corrections

1. The magnetic field exerts two forces on the domain wall
   - Mono-pole: \( F_m = \frac{2\mu_0 M_s SH x}{r} \)
   - Dipole: \( F_d = \frac{\pi \mu_0 M_s SH \lambda x}{r^2} \)
   The mono-pole has been included in the calculations. The dipole can be included by introducing an effective field
   \[
   H_{\text{eff}} = H \left( 1 + \frac{\pi \lambda}{2r} \right).
   \]

2. Due to the curvature the energy of the domain wall is lower when the spins point to the midpoint of the wire. This effect can be introduced by changing the perpendicular anisotropy
   \[
   K_{\perp_{\text{eff}}} = K_{\perp} + \frac{A \pi}{2\lambda r} - \frac{A}{r^2}.
   \]
The Eigenmodes

Determination of the eigenmodes of the magnetisation in the wire in the micro-magnetic simulations

- applying a magnetic field $\delta$-pulse in z-direction
- this pulse contains all possible frequencies
- the out of plane direction is chosen so that the torque of the field points in the same direction as the torque of the applied current
- after this excitation the system performs a damped, free oscillation.
- the eigenmodes of the nanowire can be found by spatially resolved Discrete-Fourier-Transformation.
The Eigenmodes

Discrete Fourier transformation for a wire with $r = 45\text{nm}$ after excitation with a $\delta$-pulse.
Comparing the Model to the Numerical Results

Trace of the domain wall in phase space.
The resonance of the domain wall as a function of the current frequency for different radii.
Comparing the Model to the Numerical Results

The resonance of the domain wall as a function of the current frequency for different values of the Gilbert damping.
The resonance frequency and the damping constant as a function of the ring radius.
The resonance frequency and the damping constant as a function of the Gilbert damping.
Comparing the Model to the Numerical Results

The current induced force per wall mass as a function of the ring radius.

\[ F/m \text{ (GN/kg)} \]

\[ 1/r \text{ (1/\mu m)} \]
Comparing the Model to the Numerical Results

The current induced force per wall mass as a function of the Gilbert damping.
Experimental results have been achieved by Saitoh\(^4\) for a wire with

- cross section \(S = 3150\text{nm}^2\)
- radius \(r = 50\text{µm}\)

The measured values are

- domain wall width \(\lambda = 70\text{nm}\)
- domain wall mass \(m = (6.55 \pm 0.06) \cdot 10^{-23}\text{kg}\)
- domain wall relaxation time \(\tau = \frac{1}{2\Gamma} = (1.4 \pm 0.2) \cdot 10^{-8}\text{s}\)

From numerical calculations we obtain \(K_\perp = 76175\) J/m\(^3\).

The analytic value for the wall mass is

\[ m \approx 10^{-23}\text{kg}. \]

From the experimental values of \(\tau\) and \(m\) we can calculate the Gilbert damping

\[ \alpha = 0.0114 \pm 0.0017, \]

which is in very good agreement with other experiments.

• the current driven oscillation of domain-walls has been analysed
  • with analytic approximations
  • numerically

• some geometric corrections to the harmonic oscillator model have been introduced

• the eigenmodes have been determined by SR-DFT

• the current induced domain wall oscillations have been simulated

• we found good agreement between analytical calculations, numerical calculations and experiment