Coulomb Blockade and Coulomb staircase

Zhang Jun
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Introduction

- Single-electron effect in mesoscopic world
- Dimension less than ~30nm
- Main component of single-electronics: tunnel junction with a very small capacitance and a resistance
Realization of these junctions

- Metal-insulator-metal structure
- GaAs quantum dots
- Silicon structures
- Large molecules with conducting cores
Coulomb island
Change of voltage across the junction

- The tunneling of only one electron may produce a noticeable change $e/C$ of the voltage across the junction.

- Coulomb blockade: suppression of tunneling at voltages $|V|<e/C$
Coulomb blockade

- Capacitor charge
  \[ Q_1 = C_1 V_1 \]
  \[ Q_2 = C_2 V_2 \]

- The charge on the island,
  \[ Q = Q_2 - Q_1 = -ne \]
  \[ n = n_1 - n_2 \]
Voltage drops of the tunneling junction

- $V_1 = \frac{(C_2 V_a + ne)}{C_{eq}}$
- $V_2 = \frac{(C_1 V_a - ne)}{C_{eq}}$
- $C_{eq} = C_1 + C_2$
Electrostatic energy stored in the capacitors

\[ E_s = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} \]

\[ E_s = \frac{(C_1C_2V_a^2 + Q^2)}{2C_{eq}} \]
The work done by the voltage source

\[ W_s = \int dt V_a I(t) = V_a \Delta Q \]

One electron tunneling through 2 so that \( n_2' = n_2 + 1 \), \( n' = n - 1 \), \( Q' = Q + e \), \( V_1' = V_1 - e/C_{eq} \), \( \Delta Q = -eC_1/C_{eq} \)

Result: \( W_s(n2) = -n_2eV_aC_1/C_{eq} \)
\( W_s(n1) = -n_1eV_aC_2/C_{eq} \)
The total energy

\[ E(n_1,n_2) = E_s - W_s = \frac{(C_1 C_2 V a^2 + Q^2)}{2C_{eq}} + eV_a (C_1 n_2 + C_2 n_1)/C_{eq} \]
The change in energy of the system with a particle tunneling

\[ \Delta E_2^+ = E(n_1, n_2) - E(n_1, n_2-1) \]
\[ = \frac{Q^2}{2C_{eq}} - \frac{(Q+e)^2}{2C_{eq}} + eV_aC_1/C_{eq} \]
\[ = e/C_{eq} \left[ -\frac{e}{2} + (en + V_aC_1) \right] \]
When the island is initially neutral (n=0)

\[ \Delta E_2 = -\frac{e^2}{2C_{eq}} + eV_a\frac{C_1}{C_{eq}} > 0 \]

For \( C_1 = C_2 = C \), the requirement becomes simply \( |V_a| > \frac{e}{C_{eq}} \)
Band diagram
Coulomb staircase

\[ I \left( \frac{e}{2R_f C} \right) \]

\[ V_a \left( \frac{e}{2C} \right) \]

- Without
- Including charging effects
Thank You!